International J.Math. Combin. Vol.2(2016), 144-152

Cycle and Armed Cap Cordial Graphs

A.Nellai Murugan

(Department of Mathematics , V.O.Chidambaram College, Tamil Nadu, India)

P.Iyadurai Selvaraj

(Department of Computer Science, V.O.Chidambaram College, Tamil Nadu, India)

E-mail: anellai.vocc@gmail.com, iyaduraiselvaraj@gmail.com

Abstract: Let G = (V,E) be a graph with p vertices and q edges. A *Cap* ($\wedge$) *cordial labeling* of a Graph G with vertex set V is a bijection from V to 0,1 such that if each edge uv is assigned the label

$$f(uv) = \begin{cases} 1, & \text{if } f(u)=f(v)=1, \\ 0, & \text{otherwise.} \end{cases}$$

with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. Otherwise, it is called a *Smarandache* $\wedge$ *cordial labeling* of G. A graph that admits a $\wedge$ cordial labeling is called a $\wedge$ cordial graph (CCG). In this paper, we proved that cycle C_n (n is even), bistar $B_{m,n}$, $P_m \odot P_n$ and Helm are $\wedge$ cordial graphs.

Key Words: Cap cordial labeling, Smarandache $\wedge$ cordial labeling, Cap cordial graph.

AMS(2010): 05C78.

§1. Introduction

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e = \{uv\}$ of vertices in E is called an edge or a line of G. In this paper, we proved that Cycle C_n (n : even), Bi-star $B_{m,n}$, $P_m \odot P_n$ and Helm are $\wedge$ cordial graphs.

§2. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A $\wedge$ (cap) cordial labeling of a Graph G

[1]Received February 10, 2015, Accepted December 15, 2015.

with vertex set V is a bijection from V to (0, 1) such that if each edge uv is assigned the label

$$f(uv) = \begin{cases} 1, & \text{if } f(u) = f(v) = 1 \\ 0, & \text{otherwise.} \end{cases}$$

with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. Otherwise, it is called a *Smarandache $\wedge$ cordial labeling* of G.

The graph that admits a $\wedge$ cordial labeling is called a $\wedge$ cordial graph (CCG). we proved that cycle C_n (n is even), bistar $B_{m,n}$, $P_m \odot P_n$ and Helm are $\wedge$ cordial graphs

Definition 2.1 *A graph with sequence of vertices $u_1, u_2, \cdots, u_n$ such that successive vertices are joined with an edge, P_n is a path of length $n-1$.*

The closed path of length n is Cycle C_n.

Definition 2.2 *A $P_m \odot P_n$ graph is a graph obtained from a path P_m by joining a path of length P_n at each vertex of P_m.*

Definition 2.3 *A bistar is a graph obtained from a path P_2 by joining the root of stars S_m and S_n at the terminal vertices of P_2. It is denoted by B_m, n.*

Definition 2.4 *A Helm graph is a graph obtained from a Cycle C_n by joining a pendent vertex at each vertex of on C_n. It is denoted by $C_n \odot K_1$.*

§3. Main Results

Theorem 3.1 *A cycle C_n (n : odd) is a $\wedge$ cordial graph*

Proof Let $V(C_n) = \{u_i : 1 \leq i \leq n\}$, $E(C_n) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \bigcup (u_1 u_n)\}$. A vertex labeling $f : V(C_n) \to \{0, 1\}$ is defined by

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n-1}{2}, \\ 1, & \frac{n+1}{2} \leq i \leq n \end{cases}$$

with an induced edge labeling $f^*(u_1 u_n) = 0$,

$$f^*(u_i u_{i+1}) = \begin{cases} 0, & 1 \leq i \leq \frac{n-1}{2}, \\ 1, & \frac{n+1}{2} \leq i \leq n-1, \end{cases}$$

Here $V_0(f) + 1 = V_1(f)$ and $E_0(f) = E_1(f) + 1$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1, \quad |E_0(f) - E_1(f)| \leq 1.$$

Hence, C_n is $\wedge$ cordial graph. □

For example, C_7 is $\wedge$ cordial graph as shown in the Figure 1.

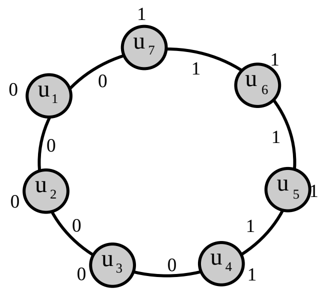

Figure 1 Graph C_7

Theorem 3.2 *A star S_n is a $\wedge$ cordial graph.*

Proof Let $V(S_n) = \{u, u_i : 1 \leq i \leq n\}$ and $E(S_n) = \{(uu_i) : 1 \leq i \leq n\}$. Define $f : V(S_n) \to 0, 1$ with vertex labeling as follows:

Case 1. If n is even, then $f(u) = 1$,

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2} + 1 \leq i \leq n \end{cases}$$

and an induced edge labeling

$$f^*(uu_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2} + 1 \leq i \leq n. \end{cases}$$

Here $V_0(f) + 1 = V_1(f)$ and $E_0(f) = E_1(f)$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1 \text{ and } |E_0(f) - E_1(f)| \leq 1.$$

Case 2. If n is odd, then $f(u) = 1$,

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n+1}{2}, \\ 1, & \frac{n+3}{2} \leq i \leq n \end{cases}$$

and with an induced edge labeling

$$f^*(uu_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n+1}{2}, \\ 1, & \frac{n+3}{2} \leq i \leq n. \end{cases}$$

Here $V_0(f) = V_1(f)$ and $E_0(f) = E_1(f) + 1$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1 \text{ and } |E_0(f) - E_1(f)| \leq 1.$$

Hence, S_n is $\wedge$ cordial graph. □

For example, S_5 and S_6 are cordial graphs as shown in the Figures 2 and 3.

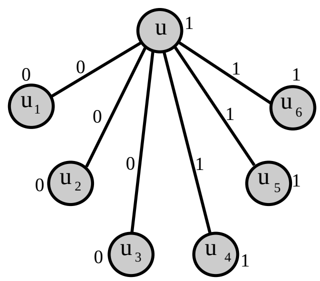

Figure 2 Graph S_6

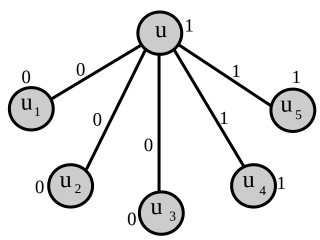

Figure 3 Graph S_5

Theorem 3.3 *A bistar $B_{m,n}$ is a $\wedge$ cordial graph.*

Proof Let $V(B_{m,n}) = \{(u, v), (u_i : 1 \leq i \leq m), (v_j : 1 \leq j \leq n)\}$ and $E(B_{m,n}) = \{[(uu_i) : 1 \leq i \leq m] \bigcup [(vv_i) : 1 \leq i \leq m] \bigcup [(uv)]\}$. Define $f : V(B_{m,n}) \to \{0, 1\}$ by two cases.

Case 1. If $m = n$, the vertex labeling is defined by $f(u) = \{0\}$, $f(v) = \{1\}$, $f(u_i) = \{0, 1 \leq i \leq m\}$, $f(v_i) = \{1, 1 \leq i \leq m\}$ with an induced edge labeling $f^*(uu_i) = \{0, 1 \leq i \leq m\}$, $f^*(vv_i) = \{1, 1 \leq i \leq m\}$ and $f^*(uv) = 0$. Here $V_0(f) = V_1(f)$ and $E_0(f) = E_1(f) + 1$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1 \text{ and } |E_0(f) - E_1(f)| \leq 1.$$

Case 2. If $m < n$, the vertex labeling is defined by $f(u) = \{0\}$, $f(v) = \{1\}$, $f(u_i) = \{0, 1 \leq i \leq m\}$, $f(v_i) = \{1, 1 \leq i \leq m\}$,

$$f(v_{m+i}) = \begin{cases} 1, & i \equiv 1 \, mod \, 2, \\ 0, & i \equiv 0 \, mod \, 2, \qquad 1 \leq i \leq n - m, \end{cases}$$

with an induced edge labeling $f^*(uu_i) = \{0, 1 \leq i \leq m\}$, $f^*(vv_j) = \{1, 1 \leq j \leq m\}$, $f^*(uv) = 0$,

$$f^*(vvm+i) = \begin{cases} 1, & i \equiv 1 \; mod\; 2, \\ 0, & i \equiv 0 \; mod\; 2, \quad 1 \leq i \leq n-m. \end{cases}$$

Here, if $n-m$ is odd, then $V_0(f)+1 = V_1(f)$ and $E_0(f) = E_1(f)$; if $n-m$ is even, then $V_0(f) = V_1(f)$ and $E_0(f) = E_1(f)+1$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1 \text{ and } |E_0(f) - E_1(f)| \leq 1.$$

Case 3. If $n < m$, by substituting m by n and n by m in Case 2 the result follows.

Hence, $B_{m,n}$ is a $\wedge$ cordial graph. □

For example $B_{3,3}$, $B_{2,6}$ and $B_{6,2}$ are cordial graphs as shown in the Figures 4, 5 and 6.

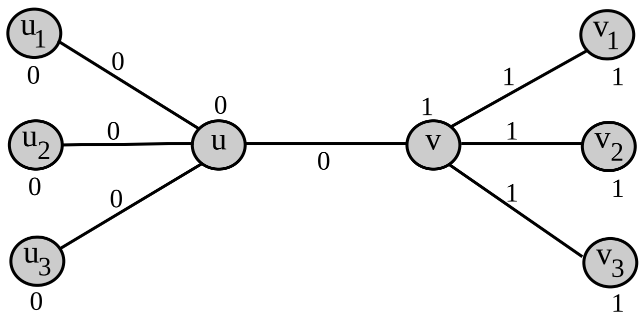

Figure 4 Graph $B_{3,3}$

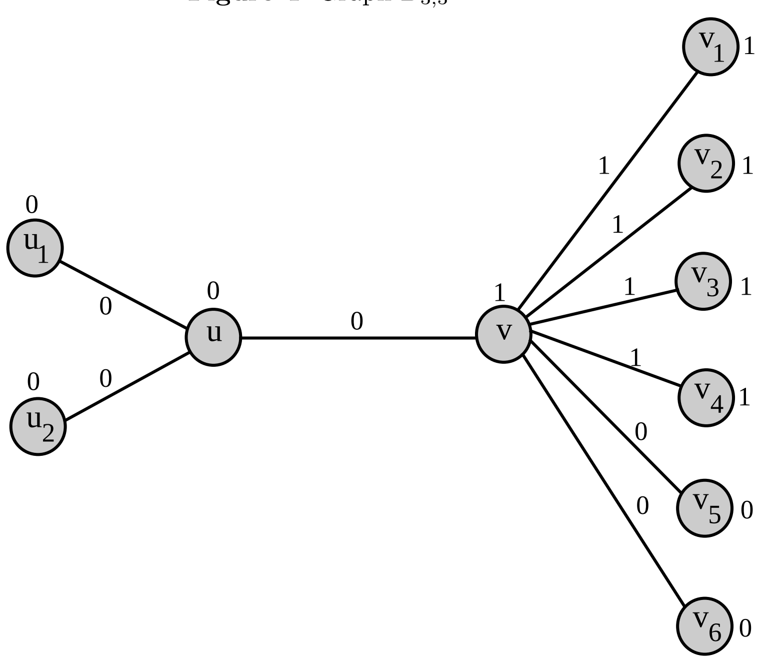

Figure 5 Graph $B_{2,6}$

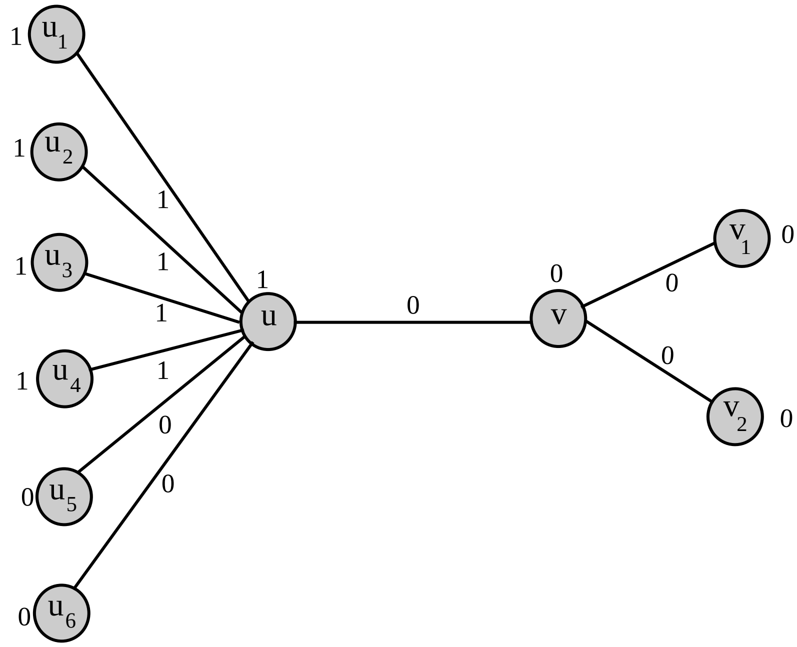

Figure 6 Graph $B_{6,2}$

Theorem 3.4 *A graph $P_m \ominus P_n$ is $\wedge$ cordial.*

Proof Let G be the graph $P_m \ominus P_n$ with $V(G) = \{[u_i : 1 \leqslant i \leqslant m], [v_{i_j} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n-1]\}$ and $E(G) = \{[(u_iu_{i+1}) : 1 \leqslant i \leqslant m-1] \bigcup [(u_iv_{i1}) : 1 \leqslant i \leqslant m] \bigcup [(v_{ij}v_{ij+1}) : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n-2]\}$. Define $f : V(G) \rightarrow \{0, 1\}$ by cases following.

Case 1. If m is even, then the vertex labeling is defined by

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq \frac{m}{2}, \\ 1, & \frac{m}{2}+1 \leq i \leq m, \end{cases} \qquad f(v_{ij}) = \begin{cases} 0, & 1 \leq i \leq \frac{m}{2}, 1 \leqslant j \leqslant n-1, \\ 1, & \frac{m}{2}+1 \leq i \leq m, 1 \leqslant j \leqslant n-1 \end{cases}$$

with an induced edge labeling

$$f^*(u_iu_{i+1}) = \begin{cases} 0, & 1 \leq i \leq \frac{m}{2}, \\ 1, & \frac{m}{2}+1 \leq i \leq m-1, \end{cases} \qquad f^*(u_iv_{i1}) = \begin{cases} 0, & 1 \leq i \leq \frac{m}{2}, \\ 1, & \frac{m}{2}+1 \leq i \leq m, \end{cases}$$

$$f^*(v_{ij}v_{ij+1}) = \begin{cases} 0, & 1 \leq i \leq \frac{m}{2}, 1 \leqslant j \leqslant n-2, \\ 1, & \frac{m}{2}+1 \leq i \leq m, 1 \leqslant j \leqslant n-2. \end{cases}$$

Here $V_0(f) = V_1(f)$ and $E_0(f) = E_1(f) + 1$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1 \text{ and } |E_0(f) - E_1(f)| \leq 1.$$

Case 2. If m is odd and n is odd, the vertex labeling is defined by

$$f(u_i)=\begin{cases}0, & 1\le i\le \frac{m-1}{2},\\ 1, & \frac{m+1}{2}\le i\le m,\end{cases}\qquad f(v_{ij})=\begin{cases}0, & t1\le i\le \frac{m-1}{2}, 1\leqslant j\leqslant n-1,\\ 1, & \frac{m+1}{2}\le i\le m, 1\leqslant j\leqslant n-1,\end{cases}$$

$$f(v_{\frac{m+1}{2}j})=\begin{cases}1, & 1\le j\le \frac{n}{2},\\ 0, & \frac{n}{2}+1\leqslant j\leqslant n-1\end{cases}$$

with an induced edge labeling

$$f^*(u_iu_{i+1})=\begin{cases}0, & 1\le i\le \frac{m-1}{2},\\ 1, & \frac{m+1}{2}+1\le i\le m-1,\end{cases}\qquad f^*(u_iv_{i1})=\begin{cases}0, & 1\le i\le \frac{m-1}{2},\\ 1, & \frac{m+1}{2}+1\le i\le m,\end{cases}$$

$$f^*(v_{ij}v_{ij+1})=\begin{cases}0, & 1\le i\le \frac{m-1}{2}, 1\leqslant j\leqslant n-2,\\ 1, & \frac{m+3}{2}\le i\le m, 1\leqslant j\leqslant n-2,\end{cases}$$

$$f^*(v_{\frac{m+1}{2}j}v_{\frac{m+1}{2}j+1})=\begin{cases}1, & 1\le j\le \frac{n-3}{2},\\ 0, & \frac{n-1}{2}\le j\leqslant n-2.\end{cases}$$

Here $V_0(f)+1=V_1(f)$ and $E_0(f)=E_1(f)$. It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and } |E_0(f)-E_1(f)|\le 1.$$

Case 3. If m is odd and n is even, the vertex labeling is defined by

$$f(u_i)=\begin{cases}0, & 1\le i\le \frac{m-1}{2},\\ 1, & \frac{m+1}{2}\le i\le m,\end{cases}\qquad f(v_{ij})=\begin{cases}0, & 1\le i\le \frac{m-1}{2}, 1\leqslant j\leqslant n-1,\\ 1, & \frac{m+1}{2}\le i\le m, 1\leqslant j\leqslant n-1\end{cases}$$

with an induced edge labeling

$$f^*(u_iu_{i+1})=\begin{cases}0, & 1\le i\le \frac{m-1}{2},\\ 1, & \frac{m+1}{2}+1\le i\le m-1,\end{cases}\qquad f^*(u_iv_{i1})=\begin{cases}0, & 1\le i\le \frac{m-1}{2},\\ 1, & \frac{m+1}{2}+1\le i\le m,\end{cases}$$

$$f^*(v_{ij}v_{ij+1})=\begin{cases}0, & 1\le i\le \frac{m-1}{2}, 1\leqslant j\leqslant n-2,\\ 1, & \frac{m+1}{2}\le i\le m, 1\leqslant j\leqslant n-2,\end{cases}$$

$$f^*(v_{\frac{m+1}{2}j}v_{\frac{m+1}{2}j+1})=\begin{cases}1, & 1\le j\le \frac{n-4}{2},\\ 0, & \frac{n-2}{2}\le j\leqslant n-2.\end{cases}$$

Here $V_0(f)=V_1(f)$ and $E_0(f)=E_1(f)+1$. It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and } |E_0(f)-E_1(f)|\le 1.$$

Hence, the graph $P_m \ominus P_n$ is $\wedge$ cordial. $\square$

For example, $P_4 \ominus P_5$, $P_5 \ominus P_5$ and $P_5 \ominus P_6$ are $\wedge$ cordial as shown in Figures 7, 8 and 9.

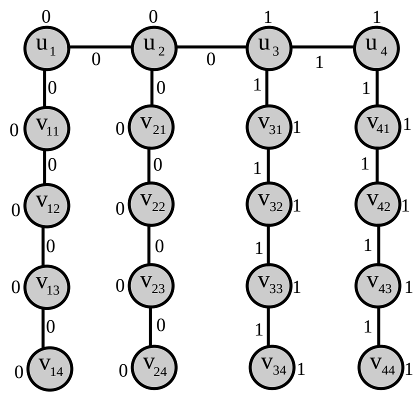

Figure 7 Graph $P_4 \ominus P_5$

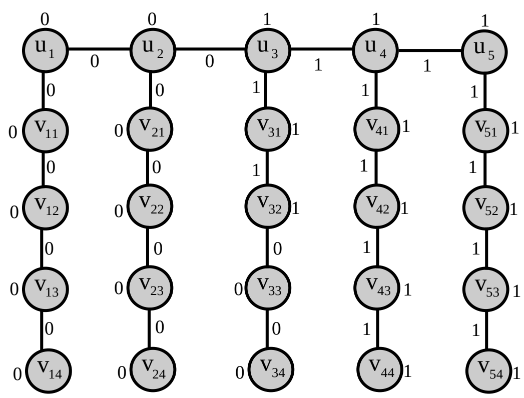

Figure 8 Graph $P_5 \ominus P_5$

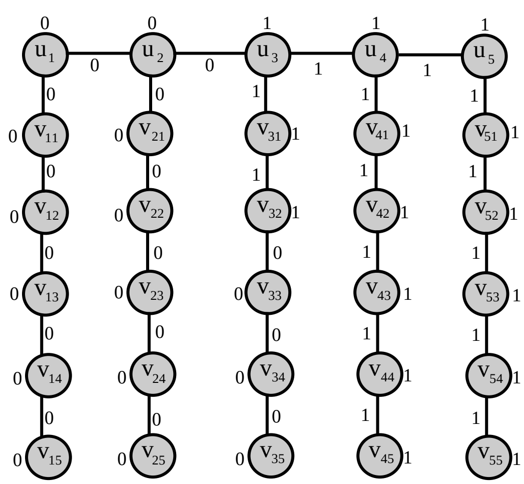

Figure 9 Graph $P_5 \ominus P_6$

Theorem 3.5 *A Helm* $(C_n \odot K_1)$ *is* $\wedge$ *cordial.*

Proof Let G be the graph $(C_n \odot K_1)$ with $V(G) = \{u_i, v_i : 1 \leqslant i \leqslant m\}$ and $E(G) = \{(u_i v_i) : 1 \leqslant i \leqslant m\}$. A vertex labeling on G is defined by $f(u_i) = \{1, 1 \leqslant i \leqslant m\}$, $f(v_i) = \{0, 1 \leqslant i \leqslant m\}$ with an induced edge labeling $f^*(u_i u_{i+1}) = \{1, 1 \leqslant i \leqslant m-1\}$, $f^*(u_m u_1) = 1$, $f^*(u_i v_i) = \{0, 1 \leqslant i \leqslant m\}$. Here $V_0(f) = V_1(f)$ and $E_0(f) = E_1(f)$. It satisfies the condition

$$|V_0(f) - V_1(f)| \leq 1 \text{ and } |E_0(f) - E_1(f)| \leq 1.$$

Hence, A Helm is $\wedge$ cordial. □

For example, a Helm $(C_6 \odot K_1)$ is $\wedge$ cordial as shown in the Figure 10.

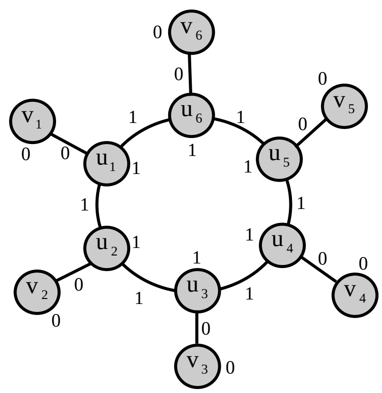

Figure 10 Graph $(C_6 \odot K_1)$

References

[1] J.A.Gallian, A Dynamic Survey of graph labeling, *The Electronic journal of Combinatorics*, 6(2001), #DS6.

[2] F.Harary, *Graph Theory*, Addition - Wesley publishing company Inc, USA, 1969.

[3] A.Nellai Murugan and V.Baby Suganya, Cordial labeling of path related splitted graphs, *Indian Journal of Applied Research*, Vol.4, Issue 3, Mar. 2014, 1-8.

[4] A.Nellai Murugan and M.Taj Nisha, A study on divisor cordial labelling of star attached paths and cycles, *Indian Journal of Research*, Vol.3, Issue 3, Mar. 2014, 12-17.

[5] A.Nellai Murugan and V.Brinda Devi, A study on path related divisor cordial graphs, *International Journal of Scientific Research*, Vol.3, Issue 4, April. 2014, 286 - 291.

[6] A.Nellai Murugan and A.Meenakshi Sundari, On Cordial Graphs, *International Journal of Scientific Research*, Vol.3, Issue 7, July. 2014, 54-55.

[7] A.Nellai Murugan and P.Iyadurai Selvaraj, Path related cup cordial graphs, *Indian Journal of Applied Research*, Vol. 4, Issue 8, August 2014.